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Developing interactive mathematical talk: investigating student perceptions and accounts of mathematical reasoning in a changing classroom context

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ABSTRACT
In recent years there has been an increased focus on the need for teachers to develop learning communities where all students have opportunities to engage in interactive discourse. However, there are few studies that focus on student perceptions and accounts of mathematical reasoning in classrooms with interactive mathematical talk as a focus of reform. A framework of teacher actions to develop classroom and mathematical practices was developed from classroom observations. Photo-elicitation interviews were used to investigate student perception and accounts of mathematical explanations and reasoning. The professional development programme, shifts in the teacher actions, and subsequent shifts in student perception and their recall of their own and peers' mathematical reasoning over a school year are highlighted. Developing interactive dialogue in the classroom took considerable time and attention. Facilitating change to the way students both participated and understood their obligations required constant, ongoing attention to both the classroom and mathematical practices.

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Background

In recent years in the United Kingdom and internationally there has been an increased focus in the educational research community on the need for teachers to develop learning communities where all students have opportunities to engage in interactive discourse in the classroom (eg Alexander, 2006; Howe & Abedin, 2013; Kessler-Singh & Robertson, 2016; Mercer & Dawes, 2014). In the area of mathematics education, this has led to calls in both research and curricular documents for reforms that lead to the development of classroom communities that promote students engaging in mathematical talk (Monaghan, 2005; Pratt, 2007). For example, despite shifts in policy which focus on more 'traditional' teaching methods (Mercer & Dawes, 2014), the most recent National Curriculum document for England (Department for Education, 2013) refers to the importance of spoken language in mathematics to ensure students present mathematical argumentation, justification or proof. Furthermore teachers are required to assist students 'in making their thinking clear

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to themselves as well as others' through the use of discussion (Department for Education, 2013).

Despite the calls for reform, studies (eg Burns & Myhill, 2004; Morgan, 2007; Watkins, 2016) examining student and teacher interactions in lessons have largely found little change in the nature of classroom discourse. It is well documented within research literature (eg Fisher, Frey, & Lapp, 2011; Mehan, 1979; Pape et al., 2010) that often within traditional mathematics classrooms a typical pattern of classroom discourse follows a structure of IRE: initiate–respond–evaluate. In this type of discourse the teacher initiates an interaction, often through questions that gather information and require immediate responses; the subsequent student response is then evaluated by the teacher. Consequently the teacher takes the role of the person who both holds and dispenses knowledge while the students receive the knowledge (Bell & Pape, 2012). Criticisms of this discourse pattern include that it lowers student engagement and offers limited opportunities for students to verbalise ideas, construct knowledge and develop a sense of agency (Bell & Pape, 2012; Fisher et al., 2011).

Developing classroom communities that promote interactive mathematical talk is challenging for many teachers and their students, particularly because they may not have previously experienced learning and teaching in such classrooms (McCrone, 2005; Sherin, 2002). There are also difficulties in developing shared understandings (between the research community, educators and students) of what constitutes interactive practices within the classroom (Morgan, 2007; Pratt, 2006). However, there are a range of studies (eg Chapin & O'Connor, 2007; McCrone, 2005; Mercer, 2000; Mercer & Sams, 2006; Monaghan, 2005; Reid & Zack, 2009; Sherin, 2002) that specifically investigate the pedagogical actions which teachers can take to guide the development of mathematical discourse in the classroom and facilitate active engagement of students. For example, McCrone (2005) illustrated how a teacher in a classroom with students aged 10 to 11 years old shifted students' participation in discourse from parallel conversations characterised by a lack of active listening to that of critical active participants. This was achieved by pedagogical actions such as modelling active listening and reflecting on the ideas of others. The teacher also initiated explicit discussions to emphasise the importance of active reflection and participation in mathematical discussions. Similarly, Reid and Zack (2009) described a classroom in which the students were expected to express their thinking and engage in analysing the reasoning of others. Changes in expectations by the teacher were accompanied by greater student awareness of how their peers could contribute to their understanding and how they could extend and reshape the ideas of others. Both of these studies demonstrate how teacher expectations and specific pedagogical actions can support the development of a classroom community that features interactive practices.

While a number of previous studies have focused on the existing or changing interaction patterns in the classroom or on the important role of the teacher, there are fewer studies that focus on the students' perspectives and accounts in classrooms with interactive mathematical talk as a focus of reform. For students who are inducted into reform classroom communities, there are shifts in their role as learners. Students are required to engage in ways of learning that privilege different forms of knowledge and participation (Fisher & Larkin, 2008; Hodge, 2008; Pratt, 2006). Engagement in collaborative interaction necessitates a shift away from the more traditional role of students as passive receivers of instruction to active and constructively critical participants within the classroom community. Students are required to express their own mathematical thinking and reasoning

and develop explanations. They also need to listen actively and make sense of a range of mathematical explanations provided by their peers. This involves students reflecting on and engaging in analysing the reasoning of others (McCrone, 2005; Reid & Zack, 2009). For students to identify and engage with these practices as part of their classroom mathematical activity, they need to be aware of how their peers can contribute to their understanding and how through listening and responding they can extend and reshape the ideas of others.

A number of studies (eg Cheeseman, 2008; Fisher & Larkin, 2008; Franke & Carey, 1997; Hodge, 2008; Morgan, 2007; Pratt, 2006) have recognised the need to consider student perspectives in relation to mathematics teaching and learning. Investigating student perspectives is also advocated within the United Nations *Convention on the Rights of the Child* (United Nations, 1989), which contends that children capable of forming their own views should be given the opportunity to express their views freely. Previous studies which have investigated student perceptions in relation to interactive practices report differing findings. Some of these studies (eg Fisher & Larkin, 2008; Morgan, 2007; Pratt, 2006) have found that students often do not view dialogue as a vehicle for learning. For example, a study by Pratt (2006) used video-stimulated recall to investigate students' views in relation to their role during interactive teaching. Despite the focus on interactive whole-class teaching in the classroom, his findings reported that these students viewed learning as an individual process. While the older Year Six students were able to provide examples of how listening to others supported their learning, the younger children in the study were unable to do this. Furthermore across the age range students did not appear to view talking as a means of developing learning and therefore privileged listening over talking as a form of meaning-making.

Other studies (eg Cobb, Gresalfi, & Hodge, 2009; Hodge, 2008) have found shifts in students' understanding of what it means to be competent in the mathematics classroom with differing instruction methods. A study by Hodge (2008) examined student perceptions with regard to their role in the mathematics classrooms. This researcher interviewed students after they had spent the first year in a classroom with discussion-based instruction methods and then again after they had moved to more traditionally orientated instruction in the second year. These students gave markedly different responses in regard to what it meant to be a good mathematics student in each class. In the classroom with discussion-based practices, competence focused on talking, thinking and listening. In contrast within the traditionally orientated classroom, success was associated with steps and answers.

There are also some studies (eg Cheeseman, 2008; Morgan, 2007) that focus on student perceptions and their recall of mathematical learning experiences. Morgan (2007) used video-stimulated recall to investigate student perceptions of learning experiences that were described as interactive by their teacher. This researcher found that students often recalled the teaching episode and the activity but found it difficult to discuss their learning. These students were frequently unable to recall what they were thinking or learning about. However, the author of this study does report that while the teachers described their teaching as interactive there were limited opportunities for students to talk and reflect and sustained dialogue was uncommon. A study by Cheeseman (2008) also used video-stimulated recall to probe students' recall of their mathematical thinking after an interaction with their teacher during a lesson. Cheeseman (2008) reported that in classrooms where there was an expectation that mathematical thinking would be explicitly described, a high number (85%) of students described their mathematical thinking explicitly.

While these studies have investigated aspects of student perceptions, it appears that there have been limited studies that have explored shifts in student perceptions and their recall of mathematical reasoning over time as changes are enacted in the classroom. The research reported in this paper focuses on student perspectives and accounts of mathematical reasoning in a classroom where the teacher reformed her practice to develop collaboration and new forms of productive talk following professional development. To provide a context the paper will first focus on the professional development undertaken with the teacher and the associated changes in the classroom. However, the overall focus of this paper will be the shifts in student perception in relation to their mathematical learning experiences and in their recall of their mathematical reasoning and the reasoning of their peers over a school year.

Theoretical framework

A close relationship is suggested between social processes and conceptual development within the body of work developed by social practice theorists. This is exemplified in the well-known social practice theory of Lave and Wenger (1991) in which notions of 'a community of practice' and the 'connectedness of knowing' are central. Some researchers, such as Boylan (2010), critique the use of social practice theory to analyse *usual school mathematics*. He argues that, in transmissive pedagogies, student participation in a mathematics community of practice is not possible because teachers already hold the required subject knowledge and are not learning and developing alongside the students. However, Boylan (2010) argues that when significantly different types of pedagogy are developed in the mathematics classroom then different forms of participation by students are possible. In this case Lave and Wenger's (1991) social practice theory becomes more applicable. Social practice theory was identified as an appropriate theoretical framework for the study reported in this paper as the teacher participant was engaged in ongoing exploration of how to integrate early algebra into her everyday classroom practice. This included significant reflection and changes to the classroom and mathematical practices to develop interactive discourse and greater participation of students.

In a social practice theoretical perspective, identity is developed by an individual as he/she engages in the everyday activities within a community of practice (for example, the mathematics classroom). As an individual participates, he/she learns the ways of thinking and acting which are valued by the community and thereby develop both a sense of what it means to be a member of a specific community and a sense of self in relation to the community (Boaler, Wiliam, & Zevenbergen, 2000; Hodge, 2008). From this perspective, identity is a function of participating in different communities; it is dynamic and situated rather than stable and consistent. For different students, there can be a greater or lesser sense of belonging in relation to the community. This is related to how students come to understand what it means to do mathematics in the classroom and to what extent they identify with this (Boaler et al., 2000; Cobb et al., 2009). The evolving mathematical identities of students include the ways they 'think about themselves in relation to mathematics and the extent to which they have developed a commitment to, and have come to see value in, mathematics as it is realised in the classroom' (Cobb et al., 2009, pp. 40–41).

Methodology

This paper reports on episodes drawn from a larger study, which involved a year-long professional development and classroom-based intervention focused on developing early algebraic reasoning in a mathematical community of inquiry. Although the focus of the study was on developing early algebraic reasoning, an aim was to also understand the key pedagogical strategies and classroom and mathematical practices that teachers can use to facilitate algebraic reasoning in the classroom. This included an emphasis on developing collaborative interaction and productive mathematical discourse.

Participants in this study included a group of primary teachers and their classes from a semi-rural school in the Channel Islands. Schools within the Channel Islands follow the National Curriculum for schools within England. The school in this study used the Mathematics Enhancement Programme (MEP) curriculum material, which was developed in order to improve mathematics teaching and learning in the United Kingdom by drawing on findings from the Kassel project (Burghes, 2004). This curriculum material includes resources such as lesson plans, workbooks, and online interactive resources and advocates an approach that uses whole-class discussions after partner or small-group work. The focus in this paper is on one classroom of 25 students aged seven to eight years old. The students were from predominantly middle socio-economic home environments and represented a range of ethnic backgrounds.

Data gathering included video records of classroom observations and professional development meetings, audio-recorded photo elicitation interviews (PEI), detailed field notes and classroom artefacts. The results reported in this paper draw on the data from the professional development meetings related to developing collaborative interaction and productive mathematical discourse to provide a context for the changes in the classroom. Other key sources of data include the classroom observations and the PEI.

Data from the classroom observations were analysed and used to inform and shape the subsequent design of an overall framework of teacher actions to develop early algebraic reasoning. Ongoing data analysis supported the revision of the model for professional development. Retrospective data analysis used QSR International's NVivo 10™ qualitative software programme (QSR International, 2012). This included multi-levels of coding using both parent and child nodes. The initial codes were developed from a variety of sources including research literature, the initial viewing of the video records, and the observational and reflective field notes. Repeated viewing of the videos and re-reading of the transcripts and field notes confirmed or refuted the initial hypotheses and codes and other hypotheses and codes were developed as necessary. A section of the framework – teacher actions to develop classroom and mathematical practices – is presented in this paper. This is shown in a table supported by examples near the beginning of each section and illustrates the changes happening in the classroom over the course of the study.

Photo elicitation interviews are a form of research interview that involves the use of photographs. Images in PEI can be used in different ways as visual inventories, depictions of events or to document dimensions of the social (Harper, 2002). Use of photographs in interviews can help develop rapport between the child participant and researcher and also ease the awkwardness of the interview situation (Clark-IbaNez, 2004; Meo, 2010). In this research, photographs were taken to capture images that depicted classroom events. Acting as 'medium of communication between researcher and participant' (Clark-IbaNez, 2004,

p. 1512) these images were used to provoke a participant's memory and prompt discussion. The photographs comprised two broad categories, first those taken during small-group work depicting instances of students solving tasks or second those taken during the whole-class discussion featuring students sharing their solution strategies. Between three and four students were interviewed in the afternoon following the observed lesson in the morning. The students were first asked general questions to gain their perspective on participating in the classroom and mathematical practices. They were then shown between three and four photographs and asked to describe what was happening from their perspective and what they were thinking. This included asking them to recall solution strategies and explanations provided by their peers along with their own and their groups' mathematical reasoning.

Informed consent was an important consideration in this study due to the involvement of children as research participants. Verbal and written information was provided to children and their caregivers prior to the beginning of the study and written consent was sought from both parties. The inclusion of the consent from students provided them with control and autonomy in the process. To further maintain student autonomy, they were always asked for verbal consent prior to any interviews and were also advised that they could choose whether to answer the questions or ask for the recording device to be turned off. This meant that their responses reflected their choice to participate.

Interviews were audio-taped and wholly transcribed. Data analysis used a grounded approach in which codes, categories, patterns and themes were developed. For the PEI the first level of analysis used two broad headings to categorise the focus of the photograph: recall of personal or group solution strategy and reasoning, and recall of peers' solution strategy and reasoning. Categories were then developed to code the student responses. Re-reading of the transcripts and field notes confirmed or refuted the initial hypotheses and codes and other hypotheses and codes were developed as necessary.

Findings

The findings will be presented from the three phases of the study. Data from the professional development sessions relevant to the development of collaborative interaction and interactive discourse will be presented to provide a context for the shifts in the classroom. Additionally the teacher actions in the classroom to address the classroom and mathematical practices during each phase are examined to provide a context for the shifts in student perceptions and recall of mathematical reasoning.

Initial interaction patterns in the classroom

Initially in this classroom paired work was a feature of the mathematics lessons; however, there was little focus on collaboration between the pairs of students. It was observed that while some pairs worked cooperatively on a task, others simply sat next to each other rather than working together. In a number of cases, the students focused on turn-taking and the partnerships appeared to be used simply as a support mechanism when the students were stuck.

During the whole-class discussions following paired or group work there were limited opportunities for student engagement in mathematical reasoning. Frequently students gave answers with no mathematical reasoning or responded with an answer phrased as

Table 1. Teacher actions to develop classroom and mathematical practices.

Lead explicit discussion about classroom and discourse practices
Ask students to apply their own reasoning to the reasoning of someone else
Require students working in pairs or small groups to develop a collaborative solution strategy that all can explain
Require students to explain their reasoning

a question, for example: 'Is it nine?'. Mathematical explanations during the discussions originated mainly from the teacher where on average she gave between four and seven explanations per lesson. In contrast, students developed between one and two mathematical explanations in each lesson. When a student provided a response with no reasoning the teacher would re-voice it and provide a mathematical explanation herself.

Phase One

Addressing the classroom and mathematical practices through professional development

In the first phase of professional development, research articles (eg Kazemi, 1998; Monaghan, 2005; Stein, Engle, Smith, & Hughes, 2008) were introduced as a means to develop links between theory and practice. The teachers were asked to read the articles and then use these as a tool along with prompts developed by the researcher to reflect on their existing practice. During this activity, the teacher noted the need to shift the pattern of the classroom discourse and to focus on teaching her students the skills of talking as well as listening. This included a developing awareness of the need for an equitable classroom community: 'It's getting those less able children to contribute too so that they can feel like they are part of the whole problem-solving thing.' She also noted the need to shift from an emphasis on teaching mathematics in a fun and exciting way to a focus on the deeper ideas: 'they had the classroom community thing which we have been pushing and pushing but the really deep thing wasn't making everything fun and exciting, you know that is not enough, it is getting deeper into the ideas'. These reflections show how the teacher was able to begin to use the research articles focused on classroom practices as a lens to view, notice and reflect on her own practice.

Addressing the classroom and mathematical practices in the classroom

Following the professional development, the teacher undertook specific actions to develop the classroom and mathematical practices in the classroom. An overview of these actions is shown in Table 1.

The teacher recognised the need to develop student participation by focusing student attention on forms of productive talk. Drawing on the research article by Monaghan (2005) she began in her classroom by facilitating a discussion of how to talk together successfully and invited the students to generate a set of rules for talk. Agreed rules included the need for active participation, individual responsibility to listen to others, and also group responsibility to ensure that all class members understood.

The facilitation of discourse required constant and deliberate actions from the teacher. These actions included the use of re-voicing and questioning. Re-voicing was used to introduce students to mathematical language. Questioning was employed to facilitate students reflecting on their peers' ideas through asking them whether they thought a response was

Table 2. Percentage of student responses (n = 28) when asked to recall their own or groups' solution strategy and reasoning.

Factor	Percentage
No response	25
Description of the task	36
Description of physical actions	10.5
Recollection of incorrect response	4
Recollection of answer with no reasoning	14
Recollection of solution strategy with some reasoning	10.5
Recollection of solution strategy with reasoning and justification	0

correct. An expectation was developed in the classroom that students analyse and make sense of reasoning. This was achieved by explicitly providing space for them to question their peers about their solution strategy and to agree or disagree. Additionally the teacher increasingly highlighted models of good practice to the class when she observed students engaged in these.

Student perspectives and accounts of the lesson

In this initial phase, nearly all the students interviewed (92%) indicated a positive disposition to partner or group work. However, the emphasis was placed on everyone having a go, turn-taking, and sharing their different ideas:

Tristan: We brainstormed and had turns telling our ideas.

Working with a partner was also viewed by many students as useful for helping themselves when they got stuck:

Paul: You had more help when you were stuck.

Table 2 shows the responses from students during the PEI when they were asked to recall their own and their groups' solution strategies and mathematical reasoning.

The students appeared to find it difficult to recall their own and their groups' mathematical reasoning. Student responses most commonly described the task.

Alexis: Mrs Stuart made a cake for her children and her and then we had to say how many you would get each.

Alternatively, other students did not recall their solution strategy and reasoning or described the physical actions that they took to solve the task. Of the students who had some recollection, most were only able to provide the answer that they had found without the reasoning for this answer. A small number of responses included the solution strategy and some reasoning for this:

Misty: There are two multiplications and two divisions for each sum.

Interviewer: Can you explain that a little bit more to me? …

Misty: Yeah like three times four is 12 and four times three is 12.

Interviewer: And what were the divisions that you could come up with?

Misty: Four, 12 divided by four equals three and 12 divided by three equals four.

Table 3 shows the responses from students during the PEI when they were asked to recall their peers' solution strategies and mathematical reasoning.

Table 3. Percentage of student responses ($n = 24$) when asked to recall their peers' solution strategy and reasoning.

Factor	Percentage
No response	25
Description of the task	35
Incorrect recall of solution strategy	12.5
Recollection of answer with no reasoning	12.5
Recollection of solution strategy with some reasoning	25
Recollection of solution strategy with reasoning and justification	0

Similarly, many of the students were unable to describe clearly the explanations their peers had provided during the whole-class discussion. Again students commonly responded by stating that they were unable to recall the explanation provided. Alternatively, their recollection focused on a description of the task. Some student responses incorrectly recalled the mathematical explanation that was given while others were able to recall only the answer provided. However, a group of student responses did recall their peers' explanation and some of their reasoning correctly:

Tristan: Duncan was saying that two of the multiplications were the same but actually three of them were the same.

Interviewer: And why were they the same, can you remember what he said?

Tristan: Because they were all just the other way around.

Phase Two

Addressing the classroom and mathematical practices through professional development

A key element of the professional development during this phase was to continue to prompt reflection on practice. This required that the teachers maintained their development of tools and skills for noticing relevant aspects of their practice (Ghousseini & Sleep, 2011). Along with the ongoing use of research articles, the teachers were provided with an adapted framework from Hunter (2009), which detailed classroom and mathematical practices linked to the development of algebraic reasoning and communication and participation. The framework provided the teachers with an objective lens to use when viewing video records from their own classrooms. It was also a useful tool to support them to reflect on their practices and set future goals.

The teacher actively sought potential areas to focus on from the research articles and framework and from analysing video records from her classroom. She embraced reflective change as an ongoing collaborative process drawing on the support from the learning community: 'I just don't seem to be making any headway so I would like some input because I keep trying 'could you ask any questions?' and then they are not getting it. I can't get my head around how to get them to ask the type of questions that I want them to.' She watched other members' videos and discussed the difficulties: 'It's hard isn't it, because you're so used to doing the questioning. It's sort of mentally having to stop yourself.' Similarly, she commented critically on her own practice as the group watched an excerpt from her classroom. To effect change in her classroom, the teacher trialled different strategies that she developed from a range of sources. This included the research articles and framework and suggestions from

Table 4. Teacher actions to develop classroom and mathematical practices.

Require that students indicate agreement or disagreement with part of an explanation or a whole explanation and provide mathematical reasons for this
Lead explicit discussions about ways of reasoning
Provide space for students to ask questions for clarification
Request students to add on to a previous contribution
Ask students to repeat previous contributions
Use student reasoning as the basis of the lesson
Facilitate students to examine solution strategies for similarities or differences
Require students to develop mathematical explanations which refer to the task and its context
Ask students to develop connections between tasks and representations
Require students to use different representations to develop the clarity of their explanation

members of the study group. As she trialled new strategies, she also carefully monitored the development and changes in student participation and the classroom practices.

Addressing the classroom and mathematical practices in the classroom

The teacher continued to undertake specific actions to develop the classroom and mathematical practices in the classroom. Table 4 shows the teacher actions that were introduced during this phase to further develop the classroom and mathematical practices in the classroom.

During this phase, a key shift for the teacher was her emphasis on facilitating student development of mathematical explanations. To achieve this, she used prompts such as: 'I also want you to think because I'm sitting here and I'm dead confused, how you could explain it to me. So I'm not just interested in your answer, I'm interested in you explaining it.'

A focus was given to facilitating students to develop an ethos and expectation of talking and working together so that the diverse students in the class learnt to think mathematically. This included the students developing shared understanding of a jointly constructed solution strategy while working in pairs or small groups. For example, during a lesson the teacher observed the students working in pairs and small groups and then told the class: 'some people are still at the stage where they don't trust their partner and they think they've got to write themselves'. To help build the discourse practices, the teacher engaged the students in reflecting on the ways in which they were working. She asked them to think about what went well or what they could improve on in terms of group interactions.

The expectation of collaborative interaction extended into whole-class discussions. The teacher positioned students to listen actively to their peers' reasoning and explanations and make sense of these. During whole-class discussions, she intervened to provide space for other students to ask questions or modelled how to ask a question herself. Other pedagogical actions that supported collaborative participation included asking students to apply their own reasoning to the reasoning of their peers and then provide reasoned agreement or disagreement. Students were also asked to add on and provide alternative solution strategies and then analyse whether these solution strategies were the same or different.

Student perspectives and accounts of the lesson

A notable feature of this phase was the students' growing recognition of the benefits of collaborative work including support for their own mathematical understanding and learning different strategies. Many indicated that they developed solution strategies by talking and listening with others. Importantly, questioning was beginning to be viewed as a way of supporting peers: 'if they don't understand, you could ask them questions and it will help them figure it out'.

Table 5. Percentage of student responses (n = 23) when asked to recall their own or groups' solution strategy and reasoning.

Factor	Percentage
No response	4
Description of the task	13
Description of physical actions	0
Recollection of incorrect response	4
Recollection of answer with no reasoning	22
Recollection of solution strategy with some reasoning	57
Recollection of solution strategy with reasoning and justification	0

Table 6. Percentage of student responses (n = 30) when asked to recall their peers' solution strategy and reasoning.

Factor	Percentage
No response	20
Description of the task	7
Incorrect recall of solution strategy	7
Recollection of answer with no reasoning	13
Recollection of solution strategy with some reasoning	40
Recollection of solution strategy with reasoning and justification	13

Fill in the answers.

100 – 10 = ☐☐	60 – 6 = ☐☐	20 – 2 = ☐☐
90 – 9 = ☐☐	50 – 5 = ☐☐	10 – 1 = ☐
80 – 8 = ☐☐	40 – 4 = ☐☐	110 – 11 = ☐☐
70 – 7 = ☐☐	30 – 3 = ☐☐	120 – 12 = ☐☐☐

Figure 1. Number sentence problems. Source: From MEP practice book Y2b (p. 126), by S. Hajdu (Budapest: Muszaki Publishing House, 1999).

In the latter part of phase two students remarked:

Justin: It's okay to disagree because if something's wrong, someone has got to know.

Other students referred to the necessity of convincing others, for example:

Zanthe: When somebody gets it wrong, we have to talk them into the right thing … because if you know it and your partner doesn't, you have to convince them to kind of say it's right.

There were shifts evident in the students' responses during the PEI as shown in Tables 5 and 6.

More than half of the student responses now recalled their solution strategy along with some reasoning for this. For example, Paul was asked to recall his thinking about a task involving the pattern in a series of equations shown in Figure 1.

Paul: It went up in ones and down in ones if you put them down in a column.

Interviewer: What went up and what went down?

Paul: The units went up and the tens went down.

At this stage although students more competently recalled their solution strategy or patterns they had noticed and provided some reasoning, they did not provide complete reasoning or justification for their thinking. A smaller group of student responses continued to describe the task or stated that they could not recall the instance photographed. The rest of the student responses recalled only the answer that they had found without any reasoning or incorrectly recalled the mathematical answer.

Shifts were also evident in student recollections of their peers' mathematical responses and explanations. The percentage of student responses which stated that they were unable to recall their peers' explanation decreased as did those responses which described the task or incorrectly recalled their peers' explanation. Accordingly the percentage of student responses that recalled their peers' solution strategies and explanation increased. Of these responses, some continued to recall only partially their peers' explanation, while other responses recalled the solution strategy provided along with some of the reasoning. At this phase of the study, a small group of student responses had begun to recall their peers' solution strategy along with the reasoning and justification that was provided.

Phase Three

Addressing the classroom and mathematical practices through professional development

During the last phase of the study, collaboration and discussion within the study group continued to take a key role in the professional development. The teachers had opportunities to engage in a lesson study cycle that drew together different elements of the professional development. Lesson study is based on a Japanese model of teacher development, which emphasises student learning and reflection on practice (Lewis, 1995; Stigler & Hiebert, 1999). Participating in a lesson study cycle can prompt teachers to reflect on their own approaches to the processes of teaching and learning and develop practices in ways that are meaningful within their working contexts (Stepanek & Appel, 2007).

Within this collegial community, the teacher identified conversations about classroom culture as a significant driver of changes in her classroom practice. In the final meeting she linked changes in practice to: 'the amount of time we've had working together as a three, because it's been sustained over the year and it's been backed up with the reading and the ongoing discussion between the three of us for the year'.

Addressing the classroom and mathematical practices

Table 7 shows the teacher actions that were introduced during the third and final phase. It is important to note that these actions built upon those previously established and outlined in Tables 2 and 4.

Table 7. Teacher actions to develop classroom and mathematical practices.

Sequence solution strategies to advance mathematical thinking and reasoning
Provide space for students to question for justification
Lead explicit discussion about mathematical practices
Promote use of concrete forms of justification
Require students to translate between different representations

During the final phase, discourse continued to be highlighted as a way of learning. Students were expected to explain their reasoning and an emphasis on developing collaborative explanations was maintained. A consistent expectation was established that students would work as a collaborative community. When students explained their strategy solutions during whole-class discussions, the teacher emphasised that their partners or group needed to listen carefully and support them when necessary. She made the speaker aware of peer support and facilitated the rest of the class to listen to the explanation and make sense of it while supporting everyone in the class to understand it.

Although an emphasis was placed on developing a collaborative community, the teacher continued to use pedagogical actions to ensure that students did not view this as always needing to agree with their peers. She emphasised mathematical argumentation when working with partners saying: 'I was really impressed with the discussion that was going on when you didn't agree with your partner.' This focus led to students attending to both their own thinking and the thinking of others and using mathematical reasoning to agree or disagree.

Student perspectives and accounts of the lesson

Responses during the PEI indicated that, for many students in this classroom, collaboration with peers had come to be viewed as an important aspect of mathematical learning. During group work the students explored and investigated each other's perspectives while also reflecting on their own. Misty described: 'once you've got all the ideas, you test one of the ideas then use the other idea if it doesn't work'. Explaining ideas to partners was viewed as beneficial in supporting both personal understanding and the understanding of peers. Students were also able to identify clearly how they reflected on solution strategies different from their own:

> **Juliana:** We tried the half and we got it right, but someone else got it right in a different way so then we tried to think it through on the next one.

They described how they used their peers' ideas from previous lessons:

> **Paul:** I used Caleb's ideas from yesterday for the hedgehogs, to use the double bit.

Both talking and listening were emphasised as key tools. Discussing mathematical ideas was highlighted as a way of approaching challenging tasks. Students described how if a task was less challenging they only talked a little but if it was difficult they talked a lot. Talking about how they were going to do it was identified as making it easier. Clear links were made to the reciprocal nature of talking and listening. Students explained how both explaining your ideas and listening to explanations were reflective tools. For example, the researcher asked whether talking helps to learn mathematics:

> **Jacqueline:** It helps me because, by teaching other people, it will make yourself get it more.

Similarly, another student described how listening to explanations during whole-class discussions was helpful:

> **Jasia:** It helps me learn because I'm saying is that right and then I'm thinking it in my brain.

Student responses showed how they were developing an understanding of the role they were required to take within the interactive nature of their classroom. For example, Juliana described listening to Alec's erroneous solution strategy and stated: 'I was a bit confused and I should have thought it through and asked a question.'

Accordingly, the number of student responses that were unable to recall their mathematical reasoning or recalled only the answer decreased significantly as shown in Table 8.

Many students now readily recalled their own and their group's solution strategy and mathematical reasoning. For example, Caleb was asked to recall a task in a lesson where the students explored what happened when you added and subtracted the same number to/from another number:

Caleb: We were doing things about 24 like taking away a number and then adding it to equal 24. We looked at that and thought it's like the same as just doing nothing to it except you take away and add it so you like lose something and then you find it again.

Interviewer: So can you think of another example that works?

Caleb: Twenty-four take away 12 add 12 would equal 24 because you're just taking away and then adding again to get it.

Interviewer: And does that always work?

Caleb: Yeah, because no matter what you take away, even if it's a really high number you can still add it on again and you equal the same.

Similarly, as shown in Table 9, when asked to recall a peer's solution strategy that was shared during the whole discussion only a small number of responses stated that they could not recall the explanation or only recalled the answer provided.

Many student responses recalled their peers' solution strategy along with some of the reasoning while the most common student response recalled the shared solution strategy along with the reasoning and justification for this. For example, Juliana re-explained a solution strategy provided by her peer to a word problem (eg If 28 apples were shared equally between four children how many apples would each child get?). As she re-explained this, she also provided the reasoning and justification for the explanation by referring to the context of the problem. With prompting she then compared this with her own explanation:

Table 8. Percentage of student responses (n = 35) when asked to recall their own or groups' solution strategy and reasoning.

Factor	Percentage
No response	6
Description of the task	0
Description of physical actions	0
Recollection of incorrect response	0
Recollection of answer with no reasoning	6
Recollection of solution strategy with some reasoning	34
Recollection of solution strategy with reasoning and justification	54

Table 9. Percentage of student responses (n = 37) when asked to recall their peers' solution strategy and reasoning.

Factor	Percentage
No response	5
Description of the task	0
Incorrect recall of solution strategy	0
Recollection of answer with no reasoning	8
Recollection of solution strategy with some reasoning	32
Recollection of solution strategy with reasoning and justification	55

Juliana: She said that they had 28 apples and they divided it into four baskets for each child and then they did four, counted in fours and they had seven fours so they worked out that the answer was seven.

Researcher: Is that the same as what your group did or did your group do it differently?

Juliana: We did it the same way but instead of doing it in baskets … we just drew the 28 apples and then we drew the four children and we just wrote an equation about 28 divided by four equals seven.

Discussion and implications

Both research literature and policy documents (eg Department for Education, 2013; Kessler-Singh & Robertson, 2016; Mercer & Dawes, 2014) highlight the importance of students engaging in mathematical talk. In this frame, teachers are tasked with developing interactive mathematical discourse through the use of specific actions. Despite an ongoing focus on mathematical talk (eg DfE, 1999, 2013; Pratt, 2006; Watkins, 2016), research studies focused on the interactions between teachers and students during lessons have reported little change in the nature of classroom discourse (eg Burns & Myhill, 2004; Morgan, 2007; Watkins, 2016). This study indicates the ongoing need for a focus on this area and adds to the existing research literature (eg Mercer & Sams, 2006; Monaghan, 2005; Reid & Zack, 2009; Sherin, 2002) by providing an exemplar of the key pedagogical strategies used by a teacher to develop the classroom and mathematical practices following ongoing professional development.

The explicit focus in both research literature and policy documents (eg Department for Education, 2013; Mercer & Sams, 2006; Monaghan, 2005) has often been on the role of the teacher in developing mathematical talk. This study illustrates the need to widen the focus to also address the role and perceptions of students in a changing classroom context. Clearly, the study illustrates that learning environments in which students view interactive dialogue as a way of developing both personal and collective mathematical thinking and understanding can be constructed. Excerpts of student voice from the PEI illustrate their changing participation and the development of the students' sense of what it meant to be a member of their specific classroom community. To account for the shifts in student perception and participation we return to the earlier theorising of other researchers (eg Boaler et al., 2000; Cheeseman, 2008; Cobb et al., 2009; Hodge, 2008; McCrone, 2005; Mercer & Sams, 2006; Monaghan, 2005).

Evident in this study are the challenges for practitioners in relation to changing the classroom context to include interactive dialogue. In the first phase of the study, although the teacher was enacting changes that required the students to take a different role, the students did not appear to understand what this entailed. Although interactive practices were being introduced, similar to the findings by Morgan (2007), many of the students were unable to explicitly describe their own mathematical reasoning following the lesson. Additionally, whole-class discussions were not viewed by students as an opportunity to develop understanding in the classroom. While the teacher asked different members of the classroom to share their solution strategies, the listening students did not appear to view their peers' explanations as a learning opportunity for themselves. This meant that they were unable to recall their peers' explanations or reasoning.

The results of the study suggest that addressing collaborative interaction during small-group work can lead to changes in the way that students participate. In the current study, student emphasis was initially on turn-taking or assistance when stuck rather than collaborating to construct joint understanding. Similar to what is described by Monaghan (2005), the teacher worked with her class to develop a set of group norms. Importantly, she also engaged her students in reflection on the ways in which they were working; this supported them to gain understanding of the benefits of collaboration beyond the emphasis on turn-taking. The students maintained a positive disposition to group-work throughout the study with their emphasis moving from cooperating to collaborating and developing joint solution strategies. Similarly, a focus on collaborative interaction during whole-class discussions led to shifts in student participation. The use of pedagogical actions that focused on interactive mathematics talk shifted students to become more active participants who asked questions to clarify ideas and agree and disagree with their peers' ideas.

Evidence from this study indicates that, within changing classroom contexts, students learn the ways of thinking and acting that are valued in the community as they participate in the developing practices of the classroom. Boaler and her colleagues (2000) and Hodge (2008) explain that this participation helps to develop a sense of what it means to be a member of a specific community. In the current study, as the teacher's actions led to changes in the classroom and mathematical practices, concurrent changes were noted in students' participation and their understanding of their role as a learner. With the increased emphasis on describing mathematical thinking, students developed competence at explicitly describing their thinking after lessons. This mirrors Cheeseman's (2008) finding that in classrooms where there was an expectation that mathematical thinking would be explicitly described, a high number of students were able to describe their mathematical thinking. The changed responses during the PEI can be viewed as representing a shift in the way students were participating and their understanding of their role as a learner. Student responses during this phase of the study illustrate that a number of them were now able to reconstruct their peers' explanations and some of their reasoning from whole-class discussions.

Further shifts can be identified in the final phase of the study, which indicated that many of the students had now developed a sense of what it meant to be a member of their specific classroom community. This aligns with earlier work by Boaler and her colleagues (2000) and Hodge (2008). Following the significant reform in their mathematics classroom, students had a changing sense of their obligations within the classroom. By the end of the study, students in this classroom were able to describe clearly their different roles and obligations within mathematics lessons. As Cobb et al. (2009) propose, they developed identities that were related to the micro-culture developed in the classroom. Their descriptions of collaborative discourse were aligned with what Mercer (2000) describes as exploratory talk. Similar to the students in Franke and Carey's (1997) study, these students now perceived doing mathematics as testing ideas, communicating thinking and using differing solution strategies. Collaborating with others and both talking and listening were identified by the students as key reflective tools for learning.

Of importance in changing student participation is the need for practitioners to address student views of whole-class discussions. As shown in this study, students need to view whole-class discussions as a way of developing both personal and collective mathematical thinking and understanding. Developing an appreciation of the collective enhanced the students' sense of obligation to provide mathematical reasoning, develop explanations and

provide justification for all members of the classroom (both other students and the teacher). As McCrone (2005) and Reid and Zack (2009) describe, the students' role had shifted towards being active and constructively critical participants in the classroom community. The sophistication of student accounts increased over the phases of the study and many students were able to recall the reasoning that supported the explanation and to provide justification for both their own and their peers' explanations.

Developing interactive dialogue in the classroom takes considerable time and specific attention. In this study, facilitating change to the way the students both participated and understood their obligations required constant, ongoing attention to both the classroom and mathematical practices. The results of the study indicate the importance of investigating student perceptions and accounts of classroom events using tools such as PEI.

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